

Chapter notes: 14 Further differentiation

Overview

This chapter extends the number of functions which can be differentiated. In each section there are also questions on optimisation and graphical interpretations of derivatives. We estimate that this will need eight teaching hours.

Introductory problem

By considering the cone when θ is very small and very close to 90° , it should be clear that there is some maximum volume. However, if an expression for the volume is created, it should become apparent that it cannot be differentiated using the methods from chapter 12. The worked solution is given at the end of the chapter, page 438; the idea being that students should be able to answer the question using the methods covered in the chapter.

14A Differentiating composite functions using the chain rule, p420

We have placed this section after trigonometric and exponential differentiation to avoid a common misconception that the chain rule only applies to functions of the form $(f(x))^n$. Often students learn a chain rule method without actually knowing that the chain rule is that stated in Key point 14.1.

The equation of the catenary, referred to in question 9, is normally of the form $y = e^x + e^{-x}$. This can lead to an interesting investigation of hyperbolic functions.

Hints for grade 7 questions:

9. (b) Rewrite $\frac{1}{e^{2x}}$ as e^{-2x} . When the derivative is set to zero, the result is a disguised cubic.

14B Differentiating products using the product rule, p425

Hints for grade 7 questions:

14. You will need to express $\sqrt{1+x}$ as $\frac{1+x}{\sqrt{1+x}}$.

15. (a) Apply logs to the equation $x^x = e^{f(x)}$.

14C Differentiating quotients using the quotient rule, p429

Hints for grade 7 questions:

7. You will need to express $\sqrt{x+1}$ as $\frac{x+1}{\sqrt{x+1}}$.

8. You will need to show that $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} > 0$ at $x = a$, given that $f'(a) = 0$ and $f''(a) < 0$.

Apply the quotient rule twice to find $\frac{d^2y}{dx^2}$.

14D Optimisation with constraints, p433

Hints for the grade 7 questions:

10. (a) The length of each of the other sides is $\frac{20-b}{2}$. Use Pythagoras to find the height.
- (b) It is sufficient to show that $b = \frac{20}{3}$.
11. Let the numbers be x and y , so that $x^2 + y^2 = a$. Find an expression for y in terms of x .
12. The distance is given by $\sqrt{(y-4)^2 + (x-0)^2}$, but you might like to minimise the distance squared.